Higher order parametric level statistics in disordered systems

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Higher order parametric level correlations in disordered systems with broken time–reversal symmetry are studied by mapping the problem onto a model of coupled Hermitian random matrices. Closed analytical expression is derived for parametric density–density correlation function which corresponds to a perturbation of disordered system by a multicomponent flux.

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Parametric level statistics reflects the response of the spectrum $\{E_n\}$ of complex chaotic systems to an external perturbation. A few years ago it was shown [1–3] that a system whose spectrum follows closely the universal fluctuations predicted by the random matrix theory [4] should also exhibit a universal parametric behavior. This conclusion was reached by analyzing the dimensionless autocorrelator of level velocities of electron in a disordered metallic sample with a ring topology, enclosing a magnetic flux φ. A diagrammatic perturbation technique was used in the range of fluxes $g^{-1/2} \ll \varphi \ll 1$ [1], while the opposite limit, $\varphi < g^{-1/2}$ [2], has been treated within the framework of the supersymmetry formalism [5,6]. [Here $g \gg 1$ is the dimensionless conductance]. In this particular problem, the parametric correlations take a universal form involving the rescaled parameter $X^2 = 4\pi g \varphi^2$, with $g = E_c/\Delta$, the ratio of the Thouless energy and the mean level spacing. Numerical simulations have supported the point that the universal character of parametric level statistics extends to a wider class of chaotic systems without disorder [chaotic billiards] whose Hamiltonian depends on some external parameter x. In such systems, the spectral fluctuations taken at different values of x become system–independent after rescaling, $x \to X$, which involves solely the 'generalized' dimensionless conductance $g = (4\pi)^{-1} \Delta^{-2} \langle \, (\partial E_n(x)/\partial x)^2 \, \rangle$. Along with the diagrammatic technique and the supersymmetry formalism, the parametric correlations have been studied in detail within the model of Brownian motion [7,8] and in the semiclassical limit [9,10].

A further burst of activity in the field occurred after it was realized [3,7,11–14] that the problem of parametric level correlations is identical to the ground state dynamics of the integrable many–body quantum model known as Calogero–Sutherland–Moser [CSM] system. This gave new information about CSM space–time (r,τ) correlation functions that can be obtained from parametric density–density correlators $\langle \, \nu(E,0)\nu(E',\varphi) \, \rangle$ involving only two different external parameters by mapping [3] $X^2 \to -2i\tau$, $E/\Delta \to r$. For the more general situation of higher order correlation functions the connection between CSM fermions and quantum chaotic systems has been established [14] as well by using the supersymmetry technique, however it has not led to any explicit analytical results beyond the two–point correlators due to enormous increase of number of entries in the supermatrix fields, thereby making any explicit calculations in that approach impossible. Extensions to higher order statistics can be performed by using an involved method of differential equations for quantum correlation functions proposed in the much earlier work [15].

In the present paper we address the issue of higher order parametric level statistics within the framework of the random matrix theory, by appealing to the model of coupled Hermitian random matrices [16]. The latter enables us to provide a complete information about parametric correlations of single electron level densities in the presence of the multicomponent flux perturbing a disordered system, characterized by a dimensionless conductance $g \gg 1$. To the best of our knowledge, this is the first detailed study of higher order parametric level statistics in disordered systems which adopts the conventional language of the random matrix theory.

In what follows we consider a weakly disordered system fallen in the universal (metallic) regime, $g \gg 1$, which is known [5] to be modelled by invariant ensembles of large random matrices. Assuming that the time reversal symmetry is completely broken (unitary symmetry), one can statistically describe an unperturbed single electron spectrum by Gaussian Unitary Ensemble [GUE] of large $N \times N$ random matrices H_0 distributed in accordance with the probability density $\mathcal{P}[H_0] \propto \exp\{-\operatorname{Tr} H_0^2\}$. Such a distribution $\mathcal{P}[H_0]$ induces the energy scale Δ being the mean level spacing, $\Delta = \pi(2N)^{-1/2}$. Let us now apply a Gaussian perturbation consisting of d components $\overrightarrow{\varphi}_d = (\phi_1, \ldots, \phi_d)$ which does not change the global unitary symmetry, and which drives the Hamiltonian H_0 to $H = H_0 + \sum_{k=1}^{d} \phi_k H_k$, with matrices H_k drawn from GUE: $\mathcal{P}[H_k] \propto \exp\{-\operatorname{Tr} H_k^2\}$ for $k = 1, \ldots, d$. This choice corresponds to the equal 'strength' of each component of the 'vector' perturbation $\overrightarrow{\varphi}_d$ since the average $\langle (H_k)_{\mu\nu} (H_k)_{\mu'\nu'} \rangle$ is independent of the index k. The quantity which provides the most detailed information about parametric correlations in the case of

the multicomponent perturbation $\overrightarrow{\varphi}_d$ is the correlator of level densities $\nu(E,\overrightarrow{\varphi}_\sigma) = \mathrm{Tr}\,\delta(E - H_0 - \sum_{k=1}^{\sigma}\phi_k H_k)$ taken at both different values of energy E and of σ. For this reason, we will concentrate on the dimensionless multipoint correlator

$$k_{p_0,\ldots,p_d}(\{\omega^{(0)}\},\overrightarrow{0};\ldots;\{\omega^{(d)}\},\overrightarrow{\varphi}_d) = \Delta^m \left\langle \prod_{i_0=1}^{p_0}\nu(E+\omega_{i_0}^{(0)},\overrightarrow{0})\prod_{i_1=1}^{p_1}\nu(E+\omega_{i_1}^{(1)},\overrightarrow{\varphi}_1)\ldots\prod_{i_d=1}^{p_d}\nu(E+\omega_{i_d}^{(d)},\overrightarrow{\varphi}_d)\right\rangle, \quad (1)$$

where $m = p_0+\ldots+p_d$, and the angular brackets stand for averaging over ensembles of Hermitian matrices H_k with $k=0,\ldots,d$. Equation (1) can be rewritten as a $(d+1)$ multiple matrix integral over matrices $\widetilde{H}_0 = H_0$ and $\widetilde{H}_{\sigma>0} = H_0 + \sum_{k=1}^{\sigma}\phi_k H_k$,

$$k_{p_0,\ldots,p_d} \propto \int d\widetilde{H}_0\ldots\int d\widetilde{H}_d \prod_{\sigma=0}^{d}\prod_{i_\sigma=1}^{p_\sigma}\mathrm{Tr}\,\delta\left(E+\omega_{i_\sigma}^{(\sigma)}-\widetilde{H}_\sigma\right)\exp\left\{-\mathrm{Tr}\left[\sum_{\alpha=0}^{d}\left(\phi_\alpha^{-2}+\phi_{\alpha+1}^{-2}\right)\widetilde{H}_\alpha^2 - 2\sum_{\alpha=0}^{d-1}\phi_{\alpha+1}^{-2}\widetilde{H}_\alpha\widetilde{H}_{\alpha+1}\right]\right\}, \quad (2)$$

with $\phi_0 = 1$ and $\phi_{d+1}=\infty$. [This convention is relaxed everywhere below Eq. (9)]. We notice that the strengths ϕ_k $(k=1,\ldots,d)$ of the perturbation are supposed to be small, $\phi_k \ll 1$. This is justified in the thermodynamic limit $N\to\infty$, since for Gaussian perturbation accepted above, ϕ_k are known to scale with N as $\phi_k = \pi N^{-1/2}X_k$, with X_k being the set of dimensionless parameters of order unity [17].

Our crucial observation is that Eq. (2) can be interpreted as a density–density correlator in the effective model of $(d+1)$ Hermitian random matrices coupled in a chain: Each matrix $\widetilde{H}_\alpha$ is represented by a point, and two adjacent matrices $\widetilde{H}_\alpha$ and $\widetilde{H}_{\alpha+1}$ are joined by a line if the coupling of the type $\exp\{c_\alpha\,\mathrm{Tr}\,\widetilde{H}_\alpha\widetilde{H}_{\alpha+1}\}$ is present in Eq. (2). In this situation, the joint probability density of eigenvalues of all the matrices in the chain can be deduced through the Itzykson–Zuber integral [18] making the model of random Hermitian matrices coupled in a chain to be a completely solvable. In accordance with the Eynard–Mehta theorem [16], the dimensionless correlator $k_{p_0,\ldots,p_d}$ can be represented as a determinant of the $m\times m$ block matrix, $m=p_0+\ldots+p_d$, consisting of $(d+1)\times(d+1)$ rectangular submatrices $K_{\alpha,\beta}$ with $\alpha,\beta = 1,\ldots,(d+1)$, each of them having $p_{\alpha-1}\times p_{\beta-1}$ entries [19],

$$k_{p_0,\ldots,p_d} = \mathrm{Det}\begin{pmatrix} \boxed{K_{1,1}\left(\omega_{i_0}^{(0)},\omega_{j_0}^{(0)}\right)}_{p_0\times p_0} & \boxed{K_{1,2}\left(\omega_{i_0}^{(0)},\omega_{j_1}^{(1)}\right)}_{p_0\times p_1} & \cdots & \boxed{K_{1,d+1}\left(\omega_{i_0}^{(0)},\omega_{j_d}^{(d)}\right)}_{p_0\times p_d} \\ \boxed{K_{2,1}\left(\omega_{i_1}^{(1)},\omega_{j_0}^{(0)}\right)}_{p_1\times p_0} & \boxed{K_{2,2}\left(\omega_{i_1}^{(1)},\omega_{j_1}^{(1)}\right)}_{p_1\times p_1} & \cdots & \boxed{K_{2,d+1}\left(\omega_{i_1}^{(1)},\omega_{j_d}^{(d)}\right)}_{p_1\times p_d} \\ \vdots & \vdots & \ddots & \vdots \\ \boxed{K_{d+1,1}\left(\omega_{i_d}^{(d)},\omega_{j_0}^{(0)}\right)}_{p_d\times p_0} & \boxed{K_{d+1,2}\left(\omega_{i_d}^{(d)},\omega_{j_1}^{(1)}\right)}_{p_d\times p_1} & \cdots & \boxed{K_{d+1,d+1}\left(\omega_{i_d}^{(d)},\omega_{j_d}^{(d)}\right)}_{p_d\times p_d} \end{pmatrix}. \quad (3)$$

The matrix kernels $K_{\alpha,\beta}$ in Eq. (3) are

$$K_{\alpha,\beta}(\xi,\eta) = \Delta\left[H_{\alpha,\beta}(\xi,\eta) - E_{\alpha,\beta}(\xi,\eta)\right], \quad (4)$$

where

$$H_{\alpha,\beta}(\xi,\eta) = \sum_{j=0}^{N-1}\frac{1}{h_j}Q_{\alpha,j}(\xi)\,P_{\beta,j}(\eta), \quad (5)$$

and

$$E_{\alpha,\beta}(\xi,\eta) = (w_\alpha * \ldots * w_{\beta-1})(\xi,\eta) \quad (6)$$

for $1\le\alpha<\beta\le d+1$; otherwise, $E_{\alpha,\beta}=0$. Here the partial weights w_α are

$$w_\alpha(\xi,\eta) = \exp\{-\frac{V_\alpha(\xi)+V_{\alpha+1}(\eta)}{2} + 2\phi_\alpha^{-2}\xi\eta\}, \quad (7a)$$

$$V_\alpha(\xi) = (\phi_{\alpha-1}^{-2}+\phi_\alpha^{-2})[\delta_{\alpha,1}+\delta_{\alpha,d+1}+1]\xi^2, \quad (7b)$$

(compare with the weight of the matrix model, Eq. (2)). The notation $(w_\alpha * \ldots * w_{\beta-1})(\xi,\eta)$ stands for the product of the partial weights w integrated over internal variables of that product. Two sets of orthogonal functions $P_{\alpha,j}$ and $Q_{\beta,j}$ entering Eq. (5) are determined recursively

$$P_{\alpha,j}(\xi) = \int d\eta P_{\alpha-1,j}(\eta)\,w_{\alpha-1}(\eta,\xi), \quad (8a)$$

$$Q_{\beta,j}(\xi) = \int d\eta w_\beta(\xi,\eta)\,Q_{\beta+1,j}(\eta), \quad (8b)$$

for $2\le\alpha\le d+1$ and $1\le\beta\le d$; the starting points of the recursions (8a) and (8b) are the polynomials $P_{1,j} = P_j$

and $Q_{d+1,j} = Q_j$ orthogonal with respect to a *nonlocal* weight $W(\xi,\eta) = (w_1 * \ldots * w_d)(\xi,\eta)$,

$$\int d\xi \int d\eta P_i(\xi) W(\xi,\eta) Q_j(\eta) = h_j \delta_{ij}. \tag{9}$$

Close inspection of the equations above shows that the basic orthogonal polynomials P_j and Q_j can be expressed in terms of Hermite polynomials, $P_j(\xi) = H_j(\xi)$, $Q_j(\xi) = H_j(\xi[1+\sum_{k=1}^{d}\phi_k^2]^{-1/2})$. Then, step–by–step integrations in Eqs. (8) yield

$$P_{\alpha,j}(\xi) = \frac{\prod_{k=1}^{\alpha-1}(\phi_k\sqrt{\pi})}{\left[1+\sum_{k=1}^{\alpha-1}\phi_k^2\right]^{j/2}} \mathrm{e}^{-F_\alpha(\xi)} \Phi_j\left(\frac{\xi}{C_{\alpha-1}}\right), \tag{10a}$$

$$Q_{\alpha,j}(\xi) = \frac{\prod_{k=\alpha}^{d}(\phi_k\sqrt{\pi})}{\left[1+\sum_{k=\alpha}^{d}\phi_k^2\right]^{j/2}} \mathrm{e}^{F_\alpha(\xi)} \Phi_j\left(\frac{\xi}{C_{\alpha-1}}\right), \tag{10b}$$

where we have introduced the Hermite functions [20] $\Phi_j(\xi) = \exp\left[-\xi^2/2\right] H_j(\xi)$. Also, we defined the function

$$F_\alpha(\xi) = \frac{\xi^2}{2}\left[C_{\alpha-1}^{-2} + \left(\phi_\alpha^{-2} - \phi_{\alpha-1}^{-2}\right)\right], \tag{11}$$

and the constant $C_\alpha = [1+\sum_{k=1}^{\alpha}\phi_k^2]^{1/2}$. [In order to compactify the formulas, it is agreed from now on that $\phi_{d+1} = \phi_d$, $\phi_0 = \phi_1$, $\sum_{k=\alpha}^{\beta<\alpha}(\ldots) = 0$, and $\prod_{k=\alpha}^{\beta<\alpha}(\ldots) = 1$]. One can verify that the orthogonality relation (9) is satisfied with

$$h_j = 2^j j! \sqrt{\pi}[1+\sum_{k=1}^{d}\phi_k^2]^{-j/2} \prod_{k=1}^{d}\left(\phi_k\sqrt{\pi}\right), \tag{12}$$

so that the first term in Eq. (4) is

$$\begin{aligned} H_{\alpha,\alpha}(\xi,\eta) &= \mathrm{e}^{F_\alpha(\xi)-F_\alpha(\eta)} \\ &\quad\times \sum_{j=0}^{N-1} \Phi_j\left(\frac{\xi}{C_{\alpha-1}}\right)\Phi_j\left(\frac{\eta}{C_{\alpha-1}}\right), \end{aligned} \tag{13a}$$

$$\begin{aligned} H_{\alpha<\beta}(\xi,\eta) &= \prod_{k=\alpha}^{\beta-1}\left(\phi_k\sqrt{\pi}\right) \mathrm{e}^{F_\alpha(\xi)-F_\beta(\eta)} \\ &\quad\times \sum_{j=0}^{N-1} \frac{\Phi_j\left(\frac{\xi}{C_{\alpha-1}}\right)\Phi_j\left(\frac{\eta}{C_{\beta-1}}\right)}{\left[1+\sum_{k=\alpha}^{\beta-1}\phi_k^2\right]^{j/2}}, \end{aligned} \tag{13b}$$

$$\begin{aligned} H_{\alpha>\beta}(\xi,\eta) &= \frac{1}{\prod_{k=\beta}^{\alpha-1}(\phi_k\sqrt{\pi})} \mathrm{e}^{F_\alpha(\xi)-F_\beta(\eta)} \\ &\quad\times \sum_{j=0}^{N-1} \frac{\Phi_j\left(\frac{\xi}{C_{\alpha-1}}\right)\Phi_j\left(\frac{\eta}{C_{\beta-1}}\right)}{\left[1+\sum_{k=\beta}^{\alpha-1}\phi_k^2\right]^{-j/2}}. \end{aligned} \tag{13c}$$

The second term in Eq. (4) is found from Eqs. (6) and (7),

$$\begin{aligned} E_{\alpha,\beta}(\xi,\eta) &= \frac{\prod_{k=\alpha}^{\beta-1}(\phi_k\sqrt{\pi})\,\mathrm{e}^{G_\alpha(\xi)-G_\beta(\eta)}}{\sqrt{\pi\sum_{k=\alpha}^{\beta-1}\phi_k^2}} \\ &\quad\times \exp\left\{-\frac{(\xi-\eta)^2}{\sum_{k=\alpha}^{\beta-1}\phi_k^2}\right\} \end{aligned} \tag{14}$$

for $\beta \geq \alpha+2$, while $E_{\alpha,\alpha} = 0$ and $E_{\alpha,\alpha+1} = w_\alpha$. Here the function G_α reads

$$G_\alpha(\xi) = \frac{\xi^2}{2}\left(\phi_\alpha^{-2} - \phi_{\alpha-1}^{-2}\right). \tag{15}$$

Now, we are in position to compute the matrix kernels $K_{\alpha,\beta}$ via Eqs. (4), (13) and (14) in the leading order in $N\to\infty$ and keeping $X_k = \phi_k N^{1/2}/\pi \sim \mathcal{O}(1)$ fixed. The simplest, diagonal kernel $K_{\alpha,\alpha}$ can be evaluated through the Christoffel–Darboux formula [21], supplemented by the asymptotics of Hermite functions,

$$\left\{\begin{array}{c}\Phi_{2N}(t)\\ \Phi_{2N+1}(t)\end{array}\right\} \simeq \frac{(-1)^N}{N^{1/4}\sqrt{\pi}}\left\{\begin{array}{c}\cos\left(2tN^{1/2}\right)\\ \sin\left(2tN^{1/2}\right)\end{array}\right\} \tag{16}$$

where $t \sim \Delta\mathcal{O}\left(N^0\right)$. One obtains,

$$K_{\alpha,\alpha}(\xi,\eta) = \mathrm{e}^{G_\alpha(\xi)-G_\alpha(\eta)} \frac{\sin\left[\pi\Delta^{-1}(\xi-\eta)\right]}{\pi\Delta^{-1}(\xi-\eta)}. \tag{17}$$

Two other cases, $\alpha<\beta$ and $\alpha>\beta$, demand more effort. For $\alpha<\beta$ we represent the sum for $H_{\alpha<\beta}$ in Eq. (13b) as a difference of two series, $\sum_{j=0}^{\infty}(\ldots) - \sum_{j=N}^{\infty}(\ldots)$. The first sum is exactly computable by making use of the Mehler summation formula [21]. In the thermodynamic limit, this procedure yields a term which is equal to $E_{\alpha,\beta}$ in Eq. (14), and therefore it gets canceled from the expression (4) for $K_{\alpha<\beta}$ which is completely due to the remaining sum $\sum_{j=N}^{\infty}(\ldots)$. To evaluate the latter, we replace the sum over j by an integral to get

$$\begin{aligned} K_{\alpha<\beta}(\xi,\eta) &= -\prod_{k=\alpha}^{\beta-1}\left(\phi_k\sqrt{\pi}\right)\mathrm{e}^{G_\alpha(\xi)-G_\beta(\eta)} \\ &\times \int_1^\infty d\lambda_1 \cos\{\pi\frac{\xi-\eta}{\Delta}\lambda_1\}\exp\{-\frac{\pi^2\lambda_1^2}{2}\sum_{k=\alpha}^{\beta-1}X_k^2\}. \end{aligned} \tag{18}$$

In the case $\alpha>\beta$ the large–j terms in Eq. (13c) yield the main contribution to the sum due to the factor $[1+\sum_{k=\beta}^{\alpha-1}\phi_k^2]^{j/2}$. Then, passing from summation to integration, we derive

$$\begin{aligned} K_{\alpha>\beta}(\xi,\eta) &= \frac{1}{\prod_{k=\beta}^{\alpha-1}(\phi_k\sqrt{\pi})}\mathrm{e}^{G_\alpha(\xi)-G_\beta(\eta)} \\ &\times \int_0^1 d\lambda \cos\{\pi\frac{\xi-\eta}{\Delta}\lambda\}\exp\{\frac{\pi^2\lambda^2}{2}\sum_{k=\beta}^{\alpha-1}X_k^2\}. \end{aligned} \tag{19}$$

Notice that the structure of the block matrix in Eq. (3) allows one to simultaneously suppress the prefactors of

the form $\prod_k (\ldots) \mathrm{e}^{(\ldots)}$ in Eqs. (17), (18) and (19). Having this in mind, we come down to the closed analytical determinantal expression Eq. (3) for $(p_0+\ldots+p_d)$–point density–density correlator with $K_{\alpha,\beta}$ replaced by $M_{\alpha,\beta}$,

$$M_{\alpha,\alpha}(\xi,\eta) \equiv \frac{\sin\left[\pi\Delta^{-1}(\xi-\eta)\right]}{\pi\Delta^{-1}(\xi-\eta)}, \tag{20a}$$

$$M_{\alpha<\beta}(\xi,\eta) \equiv -\int_1^\infty d\lambda_1 \cos\{\pi\frac{\xi-\eta}{\Delta}\lambda_1\} \times \exp\{-\frac{\pi^2\lambda_1^2}{2}\sum_{k=\alpha}^{\beta-1} X_k^2\}, \tag{20b}$$

$$M_{\alpha>\beta}(\xi,\eta) \equiv \int_0^1 d\lambda \cos\{\pi\frac{\xi-\eta}{\Delta}\lambda\} \times \exp\{\frac{\pi^2\lambda^2}{2}\sum_{k=\beta}^{\alpha-1} X_k^2\}. \tag{20c}$$

Equations (3) and (20) are the main result of the paper. They provide a detailed information about higher order parametric density–density correlations in the case of multiparameter perturbation of disordered system. Several particular correlators can be readily deduced from our general expression: (i) For the scalar perturbation, one obtains that $k_{p,q} = \Delta^{p+q} \langle \prod_{i=1}^p \nu(E+\omega_i,0) \prod_{j=1}^q \nu(E+\Omega_j,\phi) \rangle$ is determined by

$$k_{p,q} \equiv \mathrm{Det}\begin{pmatrix} M_{\alpha,\alpha}(\omega_i,\omega_j) & M_{\alpha<\beta}(\omega_i,\Omega_j) \\ M_{\alpha>\beta}(\Omega_i,\omega_j) & M_{\alpha,\alpha}(\Omega_i,\Omega_j)\end{pmatrix}, \tag{21}$$

where $M_{\alpha,\beta}$ are those given by Eqs. (20) with $\sum_k X_k^2 \to X^2$; (ii) By replacement [3] $\omega_i/\Delta \to r_i$, $\Omega_i/\Delta \to R_i$ and $X^2 \to -2i\tau$ in Eq. (21) one arrives at the space–time correlation function in the CSM model with a coupling $\lambda = 1$; here the coordinates $\{r_i\}$ correspond to the time $t=0$, while the $\{R_i\}$ refer to the time $t=\tau$.

In summary, we presented a random–matrix–theory treatment of the problem of higher order parametric spectral statistics in disordered systems with broken time reveral symmetry in the presence of the multiparameter perturbation. A complete analytical solution was based on the mapping the initial problem onto a model of random Hermitian matrices coupled in a chain. As a particular case of the general solution given by Eqs. (3) and (20), the multipoint parametric spectral correlator Eq. (21) for the scalar perturbation has been obtained. Together with a well established correspondence between CSM fermions and parametric level statistics, the latter expression provides an information about the space–time correlation function in the Calogero–Sutherland–Moser model of free, non–interacting fermions.

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